Decompaction modes of a two-dimensional "sandpile" under vibration: Model and experiments

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We report a series of experiments dealing with the progressive decompaction of a bidimensional pile of aluminum beads under vibration. We put forward a model that allows one to understand that the bulk decompaction originates from interaction between the lateral walls of the container and the beads. The conjugated role of the local geometry of the array, the solid friction, and the aspect ratio of the pile is established. A single dimensionless parameter is introduced, which, besides the acceleration, characterizes the decompacted state. This parameter is determined independently and consistently throughout several different experiments.

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I. INTRODUCTION

An increasing number of papers have recently been devoted to the dynamics of dry granular materials (for a comprehensive review see [1] and references therein). This class of materials displays a number of unusual phenomena such as heaping, arching effects, and size or shape segregation. These properties are a stimulating challenge to fundamental investigation. In addition, dry granular materials are of great concern in many industrial applications where the processing of grains is prevalent: pharmaceuticals, building materials, chemical engineering, etc.

Among the open questions which currently receive much attention is the behavior of a granular material under vibration. The problem has been tackled from two different and complementary standpoints. On one hand, it has been the object of several computer simulations dealing with numerical simulations of the dynamics [2,3] or the use of specific statistical models [4]. On the other hand, experimental research first dealing with real three-dimensional (3D) sandpiles (among others, [5]) has turned also to 2D or 1D model systems [6–9], which have the advantage of both providing direct insight into the local dynamical processes and allowing a direct comparison with the results of computer simulations which most frequently deal with reduced geometries. Note that a crucial question is still open, which is to know whether the 2D models contain in essence most of the physics of real dry granules. Nevertheless, it is worth noticing that characteristic features such as fluidization [5], avalanche [10], convection [11], heaping [5], size segregation, or vault effects [12], which were first observed in 3D, are also present in reduced dimension [6–8]. Among the information provided by experiments on model systems, one is of essential importance and has subsequently motivated the inflection of computer algorithms. It is the crucial role played by the solid friction between the granular materials and the boundaries of the containers. As shown both experimentally and from computer simulations [2,3], solid friction interaction is responsible for convection and subsequently heaping in vibrated dry granular materials [6,11].

Among other characteristic properties of granular materials is the decompaction process, which concerns the transition from a condensed state where no relative motion of granules is allowed toward a decompacted state where a global deformation of the granulate is possible. The onset of this transition was earlier baptized the "dilatancy threshold" by Reynolds [13]. Most of the recently published papers, either experimental or dealing with computer simulations, have pointed out the importance of the micromechanical parameters (shock elastic restitution coefficient and grain-grain or grain–lateral wall solid friction) as well as the existence of an acceleration threshold which determines the onset of relative motions of particles.

The behavior of an assembly of highly elastic and polished steel spheres in a 2D triangular shaped container [14] was previously investigated experimentally. Also, numerical simulations were designed to reproduce this system [15]. The pile, under vibration, exhibited a decompaction mode called fluidization, but of a different appearance than the decompaction mode we shall describe in this paper. In the latter case, the surface was shown to behave in a "fluidlike" manner and the relative motion of the grains was really looking like Brownian motion. The macroscopic density was shown to decrease progressively with height and reach a steady form, which was almost independent of the excitation phase. For this mode we keep the denomination "fluidization." We have shown that fluidization is due to a high restitution coefficient in such a way that a momentum wave can still travel from the bottom to the top of the heap without noticeable damping. It mainly concerns the collection of highly elastic material such as steel beads. In the present paper, we look at another mode of decompaction. In contrast, the material is very inelastic and the momentum wave due to the collision with the bottom is damped in the bulk. At the macroscopic level, the heap looks compacted *most of the time.* In Ref. [6] we showed, for highly dissipative grains and large friction between the grains and

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the boundaries, that the convection rolls show up in a granular material with some anomalous dynamics at time scales much larger than the excitation period. Here we make a step further by analyzing the decompaction process responsible for the occurrence of the rolls. The pertinent parameters which drive the process are evidenced using a simple model based on a continuum medium picture. This model is checked consistently against a series of different quantitative and qualitative experiments.

II. EXPERIMENTAL STUDY

The experimental setup, as well as the image processing technique, is similar to the one we used in previous works [6,7]. We use a carefully controlled driving mechanism for the vertical vibration and the cell contains a single layer of about 3000 metallic beads. We use a computer posed photograph procedure (CPP), which consists in an accumulation of snapshots of the piling at a constant excitation phase. The snapshots are processed to show only the centers of the beads. As an illustration, Fig. 1 shows such a CPP obtained after several minutes of shaking at 15 Hz of a regular packing of oxidized aluminum beads. If no motion is evidenced, the accumulated image shows a perfect triangular lattice. We observe that a definite portion of the array is allowed to move significantly via long range horizontal or 60° slip lines during a long lasting shaking process. If the observation is long enough, a well defined limit between regions where block motion is possible and regions where block motion is not possible shows up. This defines that we call the decompacted region. Starting from a rectangular perfect stacking of height h_0, we observed that the height of the decompacted portion increases when the excitation amplitude is increased. We measured very precisely and reproducibly (for a given cell) the height h of the lowest horizontal slip line observed on the CPP. We report in Fig. 2 the ratio $\alpha = h/h_0$ as a function of the reduced acceleration Γ obtained for two different aspect ratios of the cell S_0 and at a frequency of 15 Hz. The aspect ratio S_0 is defined to be the ratio of the original height of the cell by the horizontal size L, so that $S_0 = h_0/L$. We see that the larger the aspect ratio is or the deeper the cell is, the larger the acceleration needed to allow bulk movements in the lowest part of the pile.

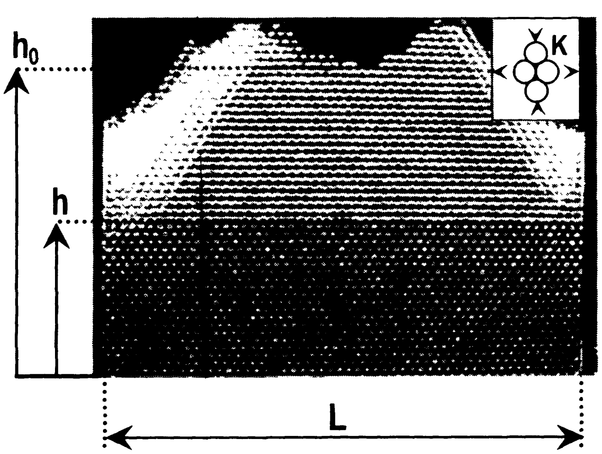

FIG. 1. Typical computer posed photograph of a vibrated 2D cell obtained at $\Gamma = 1.19$ and 15 Hz. Here the aspect ratio is $S_0 = 0.67$. The large white trails keep memory of the block motion of the heap during a 10-min experiment. The upper right corner inset illustrates the definition of the parameter K in the regular configuration.

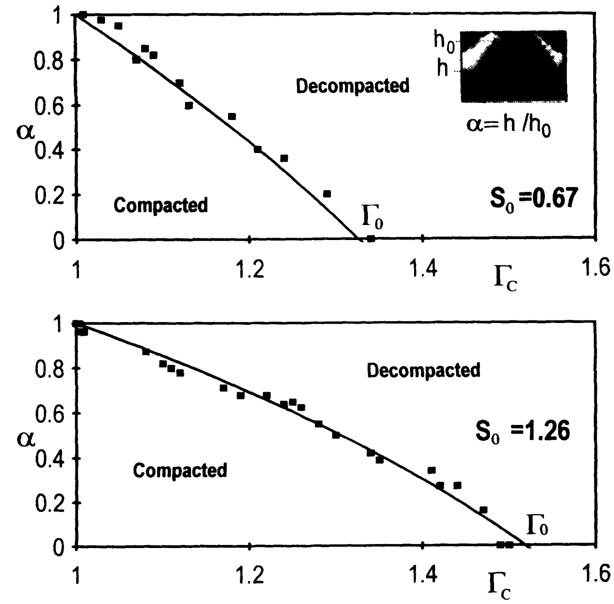

FIG. 2. Compacted-decompacted phase diagram of a vibrated 2D pile at two different aspect ratios. The cell as well as the beads are the same for both experiments. The solid squares are experimental data and full lines have been calculated from the model using $Kf = 0.29$. The inset in the upper part of the figure reminds us of the definition of α.

Figures 3(b)–3(d) give a sketch of additional experiments which were performed in order to give a hint on the pertinent parameters which govern the decompaction mechanism. Figure 3(b) has been obtained by letting under vibration, for two days, a cell ($S_0 = 1$ and $\Gamma = 2$) containing corrugated aluminum beads and exhibiting friction at the lateral boundaries. It turns out that, at the end of the experiment, the frontal windows of the glass cell have been marked by the moving beads thereby displaying a trace of the up and down motions of the successive rows of the packing. As can be observed directly on a part of the negative image reproduced in this figure, the amplitude of the motion of the beads in the framework of the cell increases with altitude. Although clearly showing the trend, this striking and simple experiment leads to rather imprecise results. But what we will show in the following is that it is yet another consistent result with our theoretical model. This experiment has been reproduced in a more quantitative manner using a miniaturized charge coupled device (CCD) camera attached to the vibrating piston, which supports the container. Doing this, and again working in the mobile referential frame, we observed that the distances between the beads were rapidly fluctuating in time. We recorded this fundamental feature by taking snapshots of the pile (Fig. 4)

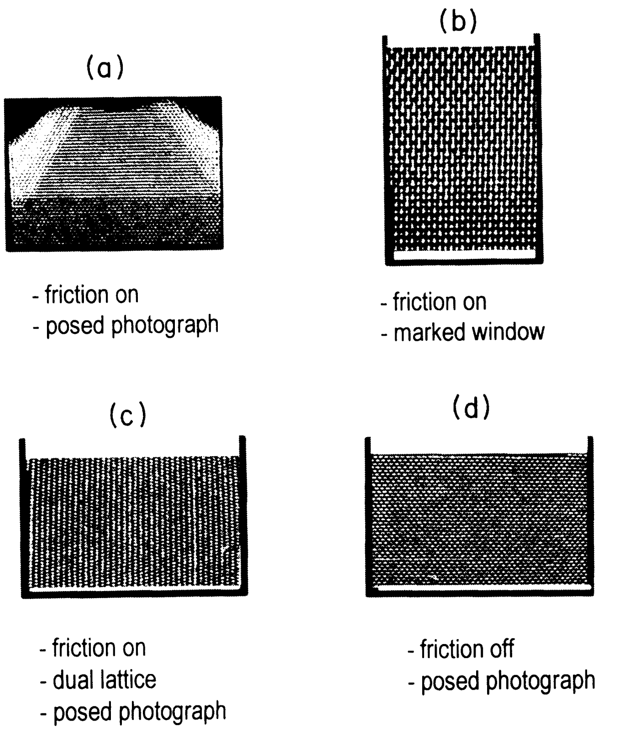

FIG. 3. Sketch of the different experiments mentioned in the text.

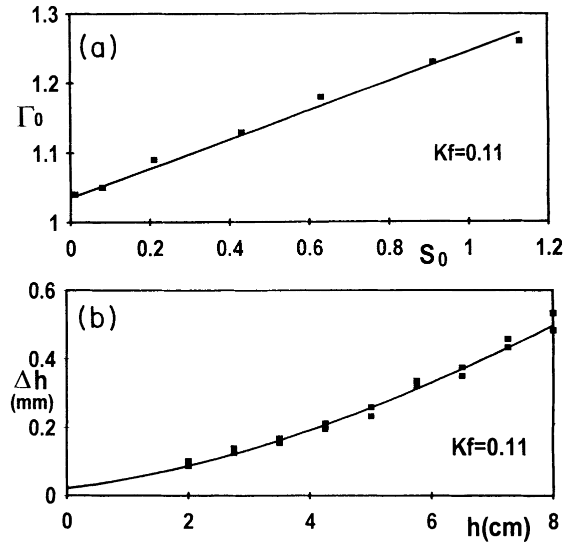

FIG. 5. (a) Takeoff accelerations as a function of aspect ratio of the same cell and beads. The solid squares are experimental data and the full line has been calculated from Eq. (10) with $Kf=0.11$. (b) Maximum elongation of the vertical motions of the beads as a function of height in the same cell as in (a). The solid squares are experimental data and the full line is obtained theoretically from the solution of Eq. (17). Here $\Gamma=1.3$.

while it was vibrated and lit from behind by a synchronous stroboscopic light. We observed that, during short fractions of a period, the stacking undergoes fast and reversible fluctuating tearings, which can be seen on the photograph as microcracks. An important question arises here which concerns the various time scales of the dynamics of the 2D sandpile as it is observed through Figs. 1–5: It is essential to remark that CPPs, as in Fig. 1, exhibit essentially the irreversible and slow motions of the beads in the container. Here the time scale of the order of more than ten periods of excitation, which is the time necessary for large scale and irreversible displacements to occur. In contrast, the time scale for inducing small and fast reversible bead motions, like the cracks reported in Fig. 4, is of the order of fraction of a period. In other words, it turns out that *the dynamics of the vibrated pile involves fast reversible fluctuating tearings (as can be seen in Fig. 4), which eventually turn out to build up constructively into slow irreversible block slip motions (as can be seen in Fig. 1).* The limiting condition for such a collective motion to occur should correspond to the Reynolds dilatancy threshold. In this paper, we leave out the detailed study of these microfluctuations, but nevertheless we note that we have found experimentally that these erratic motions seem to combine into a *smooth and regular* increase in the average decompaction amplitude as a function of the vertical position in the cell [see Figs. 3(b) and 5(b)].

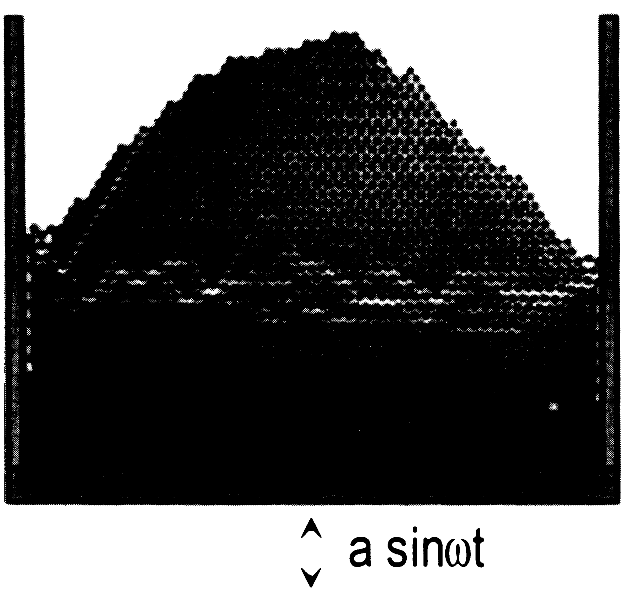

FIG. 4. Snapshot of the 2D vibrated cell showing several fluctuating and (mostly) reversible tearing lines occurring at every period during a lapse of time short compared to the period of excitation (15 Hz and $\Gamma=1.3$).

In order to discriminate between different possible mechanisms which may drive the decompaction process, we performed additional experiments reported in Figs. 3(c) and 3(d). Concerning Fig. 3(d), we performed the same experiments as in Fig. 3(a), except that we used fresh and polished aluminum beads. Within the same range of excitation parameters, namely, Γ ranging between 1 and 2, and according to our preceding findings [6], we observed neither convection nor decompaction of the pile. Under these conditions, the pile behaved as a fully compact block. Moreover, using a metallic deposit on the lateral walls, we observed that suppressing the lateral wall-bead friction led to the same result even if we used corrugated aluminum beads. These observations clearly exhibit once more the decisive role in the lateral bead-wall friction in triggering the decompaction process. This is also consistent with our previous experiments performed in a 2D cylinder where the suppression of boundaries did suppress the heaping process whereas the introduction of a stick between the windows of the cylindrical shaped annulus restored the convection [6]. Still more informative is the result of the experiments reported in Fig. 3(c), where we initially arranged the packing with a rotation $\pi/2$ compared to the triangular array configuration we usually use and which is sketched in Fig. 1 (inset). For simplicity we call this configuration the "dual" network, though we realize that this denomination may be improper. An important point to realize is that now a vertical array of beads leans on the walls instead of being on the bottom. Although these experiments, using the dual configuration, were performed with corrugated aluminum beads and lateral walls exhibiting bead-wall friction, we did not observe any convection, heaping, or decompaction. Again, the pile behaved as a block. Furthermore, for experiments performed in the spirit of Fig. 3(b), the marks on the frontal windows of the cell were found this time to be independent of the altitude. This shows that the geometry of the arrangement is essential in determining the decompaction process.

It is noteworthy that in all the preceding cases where no decompaction was observed at a moderate acceleration (say $1<\Gamma<2$), a significant increase in the excitation acceleration, above this range, would end up in starting an overall decompaction and/or fluidization process. However, as stated in the Introduction, we decided to limit our investigation within the low acceleration range where the behaviors can be easily identified and led to reproducible measurements. Staying within these limits, we will take all the preceding features into account for setting up the following simple phenomenological model.

III. A CONTINUOUS MEDIUM MODEL FOR THE DECOMPACTION OF THE 2D "SANDPILE"

A. Derivation of the compacted-decompacted phase diagram

In view of the preceding considerations and of the currently available observations, we restrict our approach to a rather simple and heuristic analysis of the behavior of the 2D sandpile. Nevertheless, as will be realized in the following, the model gives a coherent and consistent picture of the whole set of experiments. Here we work essentially in a quasistatic limit where we only consider large scale and slow motions of the pile compared to the short lived fluctuations and reversible distortions of the array evidenced in Fig. 4. In other words, the stacking is considered as a continuum and all the fast fluctuations combine into macroscopic and effective parameters. Keeping this in mind, the description of the problem has to take into account and render several essential features which have been identified all along our experiments: (i) the decisive effect of the bead-wall friction as well as the influence of the aspect ratio of the container, (ii) the progressive decompaction of the pile when the excitation amplitude is increased, and (iii) the key role played by the geometry of the stacking (regular or dual).

Now we refer to Fig. 1, which reports a static photograph of the 2D piling of aluminum beads stacked in a regular triangular array. We are interested in finding the stress distribution at the boundary of the container. We look for the quasistatic equilibrium conditions of a slice of material of width dh at a height h starting from the lower part of the cell as shown on the figure. We take into account three types of forces: (i) the bulk force exerted by the weight of the slice, (ii) the force due to the stress gradient, and (iii) the effective friction forces exerted at the frontal and lateral vertical boundaries of the container.

The magnitude of the friction forces at the walls can be determined using, first, the argument of Janssen [16], who proposed that it is a characteristic property of a granular material to be able to convert a part of the vertical component of a stress σ_{zz} into a horizontal one σ_{xx} such that $\sigma_{xx}=K\sigma_{zz}$. In our particular case of a 2D regular geometry (inset in Fig. 1), this property can be readily understood considering the diamond-shaped elementary pattern of the piling. Then, this horizontal component of the stress interacts with the lateral and frontal boundaries of the cell via the Coulombs friction. We call f the coefficient of proportionality of the Coulomb friction force defined as $\sigma_{xz}=f\sigma_{xx}$. In the following, since all the resulting forces are directed vertically, and for the sake of simplification we will make use of a single p component instead of the detailed expression of the stress tensor. Actually, the friction forces oppose the up and down motion of the considered slice of material. In the following, without any loss of generality, the direction of these forces will be taken *on the average* to be in the upward direction. It is possible to verify *a posteriori* that this choice gives a consistent picture of the stress distribution. Due to the geometry of our experimental setup and to the friction properties of the materials used, we expect that the K and f parameters will be different for frontal and lateral boundaries. Frontal boundary coefficients are K^* and f^*. We suppose an horizontal symmetry of the problem. We write now the equilibrium equation between forces per unit width:

$$-L\rho_0 g\,dh - L\frac{\partial P}{\partial h}dh + 2KfP\,dh + 2K^*f^*P\,dh = 0\ , \tag{1}$$

ρ_0 and g being, respectively, the mean density of the rows of beads and the gravitational acceleration.

Now, we estimate that frontal boundaries only play a minor role in the decompaction process, i.e., $K^*f^* \ll Kf$ because both K^* and f^* are expected to be small compared to K and f: (i) f^* stands for the bead-polished frontal windows friction coefficient, which is much smaller than the relatively high bead-plastic lateral wedges friction coefficient (f is typically 0.8). (ii) Due to the confined geometry of the 2D configuration where a single layer of particles is in contact with both frontal windows thereby prohibiting a redistribution of stress as implied by Janssen's argument (see inset in Fig. 1), K^* is expected to be vanishingly small. Also, this is consistent with the fact that no noticeable decompaction was observed when the lateral boundaries were rendered smooth and/or when the dual network was used. By integration of Eq. (1) and using the free boundary property $P(h=h_0)=0$, we obtain the effective stress distribution

$$P(h)=\frac{\rho_0 g L}{2Kf}\left[1-\exp\left[\frac{2Kf}{L}(h-h_0)\right]\right] . \quad (2)$$

This equation can be regarded as a 2D version of the oft quoted Janssen formula [16], which we recall here in order to remind the reader of the assumption used in its derivation. It means that the stress increases linearly with depth from zero exhibiting an hydrostatic profile $P(h)\cong\rho_0 g(h_0-h)$ and then saturates to a constant value $P(0)=\rho_0 g(L/2Kf)$. This is again an effect of the vaults starting from the boundaries. Thus Eq. (2) can be seen as defining a vaults range to be $L/2Kf$.

We see qualitatively that since the stress increases with the depth in the bulk of the material, we need to find a limit where no relative motion between the grain and the boundary is possible because the Coulomb threshold "pins" the grains. In the following, we express this limit quantitatively and identify the threshold with the limit of long range motions observed experimentally (see Fig. 1, for example).

We now take a vision of the system with a fine time resolution, but still we suppose that the stress distribution given by Eq. (2) is maintained all over the excitation phases. We can consider the cell as being an assembly of slabs of height dh, piled on the top of the others. If we take an excitation of the cell in the form

$$A(t)=a\sin\omega t , \quad (3)$$

a detachment may occur from a quasicompacted state in the region where $0<\omega t<\pi/2$ (2π). Then, we write the equation of dynamics for the rigid "slab" in this phase quadrant. Let us consider this slab having some upward but small relative motion $(\dot{h}\cong 0^+,\ddot{h}\cong 0^+)$. The friction forces with the boundaries oppose the upward motion and are directed downward. In the limit where no other interaction with the other slabs is considered (except through the effective stress P) we have

$$dm(\ddot{h}-a\omega^2\sin\omega t)=-dm\,g-dF_{\text{frict}} , \quad (4)$$

where

$$dm=\rho_0 L\,dh , \quad dF_{\text{frict}}=2KfP\,dh . \quad (5)$$

Then we obtained the condition necessary for a detachment of the slab as

$$\frac{\ddot{h}}{g}=\frac{a\omega^2\sin\omega t}{g}-\left[1+\frac{1}{g}\frac{\partial F_{\text{frict}}}{\partial m}\right]>0 . \quad (6)$$

We see from Eq. (6) that since $(1/g)(\partial F_{\text{frict}}/\partial m)$ is a positive function, increasing when the height is decreased, there is a limiting minimal height h_m, for which (6) can be true. It is when $\omega t=\pi/2$ (2π) and

$$\frac{a\omega^2}{g}=1+\frac{1}{g}\frac{\partial F_{\text{frict}}}{\partial m} . \quad (7)$$

Then, using relations (5) in (7), we obtain the limiting height h_t, for a possible upward motion relative to the cell. We put this in a scaled form

$$\alpha=1+(2\chi)^{-1}\ln(2-\Gamma) , \quad (8)$$

with the height ratio $\alpha=h_t/h_0$ obtained as a function of *the reduced acceleration,* which we usually define as $\Gamma=a\omega^2/g$. The only parameter that is dimensionless which we call "decompaction parameter" in the following, is

$$\chi=S_0 Kf , \quad (9)$$

which depends on the effective coefficient K of transmission between vertical and horizontal stresses σ_{zz} and σ_{xx}, the Coulomb friction f, and the aspect ratio of the cell S_0. Note two immediate consequences of (8): (i) Acceleration $\Gamma=2$ defines a limit for a granular material being stuck by the boundaries. (ii) For any block there is a limiting acceleration

$$\Gamma_0=2-\exp(-2\chi) \quad (10)$$

beyond which the whole pile will start moving.

B. Experimental tests of the decompaction diagram

In the following we test experimentally the validity of Eqs. (8)–(10). Equation (8) can be illustrated by using a phase-diagram-like picture as in Fig. 2, where we plot the functions $\alpha(\Gamma)$, which indicate the limit between decompacted and compacted domains, as a function of a reduced acceleration Γ and for two given aspect ratios S_0 (for the same cell and the same granular material). As can be seen on this figure, the experimental data fit well the theoretical model represented by solid lines using a *single* Kf parameter ($Kf=0.29$).

Now, let us consider Eq. (10). It turns out that Γ_0 can be measured experimentally quite easily and at a rather good approximation using the following procedure. We start from a null amplitude of excitation and at a given frequency; we progressively increase the amplitude of the excitation. We measure the reduced acceleration when the lowest row of the piling is seen to take off regularly from the bottom of the container. The observation is made with a stroboscopic flashing lamp slightly out of phase and through a CCD camera attached to the vibrated piston. This experiment was repeated for different S_0 values. Experimental data are reported in Fig. 5(a). We see that measured data can be fitted satisfactorily with Eq. (10) using a single characteristic parameter $Kf=0.11$. Note that it is another cell than the previous one.

Now, we check the dependence of the decompaction process on the Kf product that enters in Eq. (9). Actually we realized that different cells exhibit different Kf parameters due to the fact that there is little control of the roughness and consequently on the bead-wall friction coefficient f of different machine-finished lateral walls.

Furthermore, we observed experimentally that depositing a thin aluminum layer on the lateral walls nearly suppressed the friction coefficient with the oxidized aluminum beads. We reported earlier that, in this case, no decompaction would occur consistently with a value $f \cong 0$. Also, as expected, we got the same result by replacing oxidized aluminum beads by fresh and polished ones, which again suppressed the bead-wall friction coefficient. Now we would like to test the influence of the parameter K, which may be considered as a sort of Poisson coefficient in solids (though Poisson coefficient is, properly speaking, defined for strain ratios rather than stress ratios). Since it is an effective parameter it would be difficult to derive it from first principles, but we shall see that its action can sometimes be suppressed. Earlier we mentioned the experiment where we were using the "dual" configuration of the piling [see Fig. 3(c)] and no decompaction or heaping was occurring. This effect has an interpretation in the framework of our model. When we compare the dual configuration with the regular one, we see that all the beads touch each other along vertical lines which form a continuous and vertical network of forces (see the inset of Fig. 1, where the diamondlike figure should be rotated by 90°). Thereby it should be no surprise that, in this case, the transmission of vertical forces to horizontal forces should be minimal and thus $K \cong 0$, which again, is consistent with our model for which no decompaction occurs.

C. Study of decompaction gradients in the decompacted phase

Now, since the previous experimental tests give some confidence in the model, we shall push it a little further and check its implication by testing the amplitude of decompaction as a function of the height. Again, we consider the same analogy of the independent sets of slabs acting on the boundary with the stress gradient of Eq. (2). The vertical position of the slab is given by $z(t)=h+\Delta h(t)$, where h is the vertical position at "rest." Once the slab has some relative motion compared to the cell, it follows that

$$\ddot{z} = -g\left[2-\exp 2\chi\left[\frac{z}{h_0}-1\right]\right]+a\omega^2\sin\omega t \ . \tag{11}$$

Equation (11) is valid just in the case when $\dot{h}>0$; otherwise we would have to consider a friction force acting upwards on the pile. The relative motion starts at a time t^*, which is given by

$$\sin\varphi^* = \Gamma^{-1}\left[2-\exp 2\chi\left[\frac{h}{h_0}-1\right]\right] \tag{12}$$

with $\varphi^*=\omega t^*$ and at $t=t^*$, $\dot{h}=0$ and $\ddot{h}=0$. Equation (11) is *a priori* a nontrivial nonlinear differential equation, but we can consider that the relative motion is of small amplitude and that the stress exerted on the walls will not change much during the excitation. Thus we consider the differential equation

$$\Delta\ddot{h} = -g^* + a\omega^2\sin\omega t \ , \tag{13}$$

where

$$g^* = g\left[2-\exp 2\chi\left[\frac{h}{h_0}-1\right]\right] . \tag{14}$$

Equation (13) is then easy to integrate

$$\Delta h = -\tfrac{1}{2}g^*(t-t^*)^2 + a[\sin\omega t + \omega(t-t^*)\cos\varphi^*] \ , \tag{15}$$

$$\Delta\dot{h} = -g^*(t-t^*) + a\omega(\cos\omega t + \cos\varphi^*) \ . \tag{16}$$

We are interested in the maximum relative height. This value is obtained for a time t_{max} corresponding to $\Delta\dot{h}=0$, which leads to the implicit equation

$$g^*(t_{max}-t^*) = a\omega(\cos\omega t_{max} + \cos\varphi^*) \ . \tag{17}$$

Then we should use Eq. (15) to obtain $\Delta h_{max}=\Delta h(t_{max})$. There is a set of three implicit equations to solve [(12), (17), and (15)], which is done using a numerical routine on a computer.

Now we perform an experimental measurement of the decompaction gradients. Since the decompaction amplitudes are small, in order to optimize the measurements, we work preferentially in the domain where the acceleration is larger than Γ_0 [called takeoff acceleration in the following; see Eq. (10)] and thus the whole pile undergoes relative motion compared to the cell. We accumulate snapshots using a CPP and a video camera in the vibrating reference frame. We measure the maximal amplitude of displacement as a function of height. The results of the measurements are reported on Fig. 5(b). Introducing the parameter $Kf=0.11$, which has been found *independently* from the preceding experiment [Fig. 5(a)], which was performed in the same cell, we compute a solution from our model. We find satisfactory agreement with the experimental data reported in Fig. 5(b) *without introducing any new fitting parameter.*

IV. DISCUSSION AND CONCLUSION

The work we present here is an investigation into modes of decompaction of a dissipative sandpile. Since the internal energy dissipation between grains in contact is high, a momentum wave resulting from the shock with the bottom is strongly damped and does not fluidize the upper layers. This is a situation common to numerous grain assemblies. On the other hand, we have shown that, in this case, the lateral walls play a capital role and that the decompaction process can be rationalized *in terms of a vault effect originated from the boundaries* and propagating inside the bulk. We put forward a macroscopic model to understand more clearly these features. We could predict an effective stress gradient acting on the walls. We found this model to be both qualitatively and quantitatively consistent with a series of experiments involving a variation of the aspect ratio of the pile, the friction condition at the lateral walls, and the internal geometrical structure. We predicted and measured a sharp transition between a compacted and a decompacted region. In any case, the model shows that this separation is bound to disappear for accelerations larger than $2g$. Maybe one of the most surprising results is the pertinence of the effective stress gradient in the investigation of the short-time dynamics (decompaction amplitude averaged over a large number of observations though). Once again, the comparison with experiments is fair. We have shown that the vertical scale of relative fluctuations between the grains is increasing with height (when decompaction is mechanically possible of course). Nevertheless, in view of its relative simplicity, a brief discussion concerning the domain of validity of the picture we propose is required. First of all, one should realize that this rather crude picture of the pile neglects the time-resolved dynamics of the decompaction as it is observed in Fig. 4 (microcrack waves). Actually it deals either with time scales much larger than the period of excitation (block motion) or with quantities averaged over several periods (mean decompaction). The stress profile that is found was really tested (indirectly) in the decompacted region, thereby leading to a prediction for a threshold. However, nothing is certain about its validity inside the compacted region where another mechanical picture could take place.

Also, the simplification of a horizontally symmetric block turns out to be quite untrue in the upper part of the pile where no decompaction wave is observed. Along the same lines, we realize that our continuum model, which implies compact horizontal slices of the material, oversimplifies the role of the friction bead-bead interaction. In the real situation and using polished or corrugated beads, it is impossible to differentiate experimentally and quantitatively the respective roles of the bead-bead and bead-wall friction interactions. Under these conditions, one might ask the question of what would occur if we could turn off the bead-bead friction while turning on the bead-wall friction interaction. Moreover, an investigation of the possible dependence of the effective decompaction parameter with the frequency might be of great interest. We are aware that all the implications of this model should be tested more extensively.

One of the goals of the present work was also to push for more detailed comparisons with more sophisticated theoretical efforts as well as with computer simulations, on the grounds of firm and reproducible experimental facts.

It is worth noticing that some computer simulations have previously led to the claim of "partial fluidization." Taguchi [4] has shown that his computer simulations, in rectangular vibrated cells, exhibit the coexistence of a "fluidized" domain overlying a compact zone. A study is made that shows that the depth of the fluidized domain increases with the acceleration of the container. It is noteworthy that Taguchi's algorithm, in a certain manner, includes the conversion of vertical solicitation to horizontal ones, but in his work the motor leading to convection is some form of viscoelastic relaxation and not Coulomb friction at the walls, contrary to what we demonstrate in our present and previous works [6]. Another recent simulation work performed by Moreau [17] dealt with 2D vibrated cells similar to those we used in the present work. Using a complete mechanistic description of the problem in terms of description of shocks, Coulomb bead-bead and bead-wall friction, he produced quite realistic pictures which exhibit the progressive decompaction effect as well as the crack waves we evidence here. Further work is in progress along these lines in order to check more thoroughly the coherence of our experimental observations with Moreau's computer model.

A crucial question arises here as to whether the preceding observations and model could apply to the more complex situation of a 3D multidisperse sandpile.

Literature has occasionally reported experimental observations corroborating our findings. In particular, Laroche *et al.* reported in [5] the observation that, under moderate excitation, only the higher part of the 3D sandpile was the locus of internal motions while the bottom remained compact. Also Evesque, Szmatula, and Denis [18] reported the same observation in an experiment where a test tube filled with sand was plunged inside a sand bucket. Under vibration and for a moderate depth of the immersed boundary of the test tube, a creeping of the sand out of the tube could be observed. For deeper immersion, the motion would stop. This can be interpreted as a coexistence between two states of the sandpile, i.e., compacted and/or decompacted in the spirit of our experiments and model.

However, yet in the absence of quantitative experimental observations of the bulk decompaction of a vibrated 3D real sandpile, we cannot take for granted that all the reported features would extrapolate directly to real 3D situations. In particular, it turns out that the somewhat "crystallized" nature of our samples implies that some preferential dislocations lines (say horizontal or at 60°) are more likely to occur in 2D than they would in a polydisperse 3D sample. This particular feature probably tends to expand to a larger distance in the bulk, the perturbations induced by the friction interaction at the lateral walls. Keeping this in mind, as far as we could, we paralleled the reported quantitative experiments in 2D with rough qualitative observations in real 3D sandpiles. We also observed in 3D the formation of localized eddies at the lateral corners at acceleration just above threshold, as well as the coexistence of superimposed compacted and decompacted states. At the present time, not being in a position to see inside the bulk of the 3D sandpiles (as NMR imaging might), we have not noticed any novel and divergent behavior in 3D from the above reported results in 2D.

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[1] H. M. Jaeger and S. R. Nagel, Science **255**, 523 (1992).
[2] H. J. Hermann, Physica A **191**, 263 (1992), and references therein; G. Baumann, E. Jobs, and D. E. Wolf, Fractals **1**, 767 (1993).
[3] J. J. Moreau and M. Jean, in *Proceedings of the Contact Mechanics International Symposium*, edited by A. Curnier (Presses Polytechnique et Universitaires Romandes, Lausanne, 1992), pp. 31–48.
[4] Y.-H. Taguchi, Phys. Rev. Lett. **69**, 1367 (1992), and references therein.
[5] P. Evesque and J. Rajchenbach, Phys. Rev. Lett. **62**, 44 (1989); C. Laroche, S. Douady, and S. Fauve, J. Phys. (Paris) **50**, 699 (1989).
[6] E. Clement, J. Duran, and J. Rajchenbach, Phys. Rev. Lett. **69**, 1189 (1992).
[7] J. Duran, J. Rajchenbach, and E. Clement, Phys. Rev. Lett. **70**, 2431 (1993).
[8] J. Rajchenbach, E. Clement, and J. Duran, in *Powder and Grains 93*, edited by C. Thorton (Balkema, Rotterdam, 1993), p. 333.
[9] S. Luding, E. Clement, A. Blumen, J. Rajchenbach, and J. Duran, Phys. Rev. E **49**, 1636 (1994).
[10] J. Rajchenbach, Phys. Rev. Lett. **65**, 2221 (1990); H. M. Jaeger, C.-H. Liu, and S. R. Nagel, *ibid.* **62**, 40 (1989).
[11] J. B. Knight, H. M. Jaeger, and S. Nagel, Phys. Rev. Lett. **70**, 3728 (1993).
[12] J. C. Williams, Powder Technol. **15**, 245 (1976).
[13] O. Reynolds, Philos. Mag. Ser. 5 **20**, 469 (1885).
[14] E. Clement and J. Rajchenbach, Europhys. Lett. **16**, 133 (1991).
[15] J. A. C. Gallas, H. J. Hermann, and S. Sokolowski, Phys. Rev. Lett. **69**, 1371 (1992); Physica A **189**, 437 (1992).
[16] H. A. Janssen, Z. Vereins Deutsch Ing. **39** (35), 1045 (1895).
[17] J. J. Moreau (private communication).
[18] P. Evesque, E. Szmatula, and J. P. Denis, Europhys. Lett. **12**, 623 (1990).

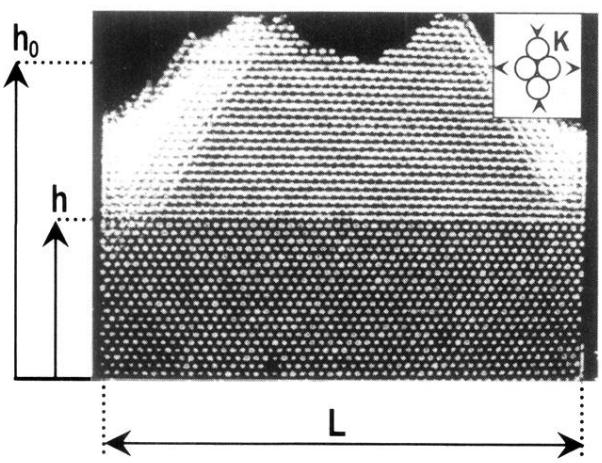

FIG. 1. Typical computer posed photograph of a vibrated 2D cell obtained at $\Gamma = 1.19$ and 15 Hz. Here the aspect ratio is $S_0 = 0.67$. The large white trails keep memory of the block motion of the heap during a 10-min experiment. The upper right corner inset illustrates the definition of the parameter K in the regular configuration.

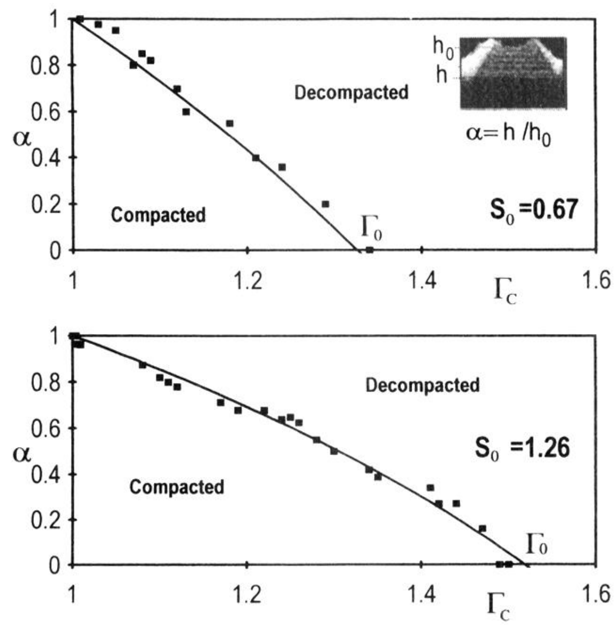

FIG. 2. Compacted-decompacted phase diagram of a vibrated 2D pile at two different aspect ratios. The cell as well as the beads are the same for both experiments. The solid squares are experimental data and full lines have been calculated from the model using $Kf = 0.29$. The inset in the upper part of the figure reminds us of the definition of α.

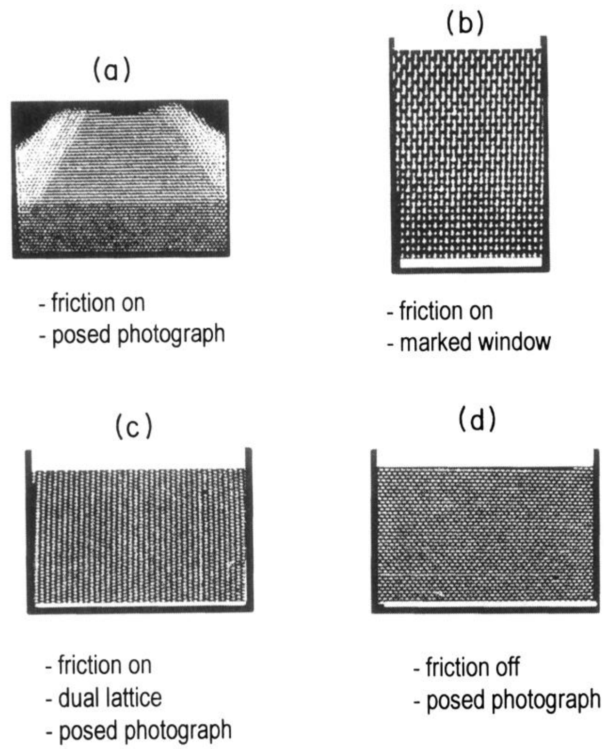

FIG. 3. Sketch of the different experiments mentioned in the text.

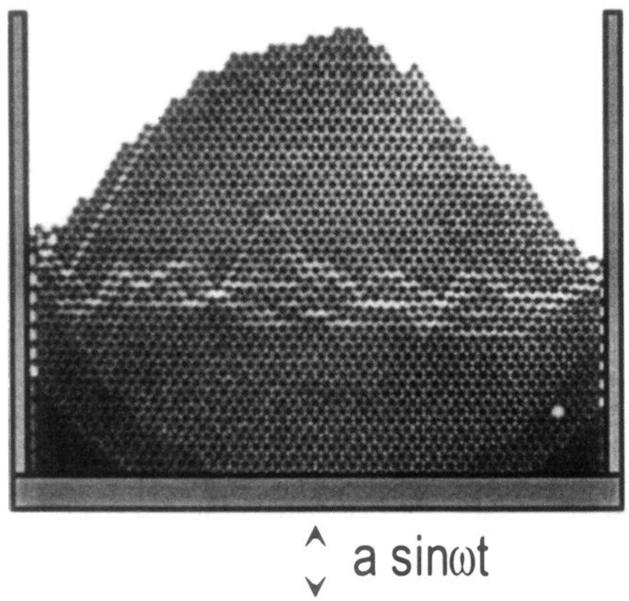

FIG. 4. Snapshot of the 2D vibrated cell showing several fluctuating and (mostly) reversible tearing lines occurring at every period during a lapse of time short compared to the period of excitation (15 Hz and $\Gamma = 1.3$).